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ANALYSIS OF TRANSISTOR CLASS A OSCILLATOR

by

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Technical Report

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in

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in the

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of the

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SUMMARY

A simple design procedure for Class A small signal oscillation has been outlined in the literature.[1, 2, 3] The procedure was verified by constructing a circuit based on this theory and comparing actual results with calculated results. An extension to large signal operation for maximum output power also appears in the same literature. This theory was checked to see if modification was necessary. It was found that a simple piecewise linear approximation to the emitter diode characteristic, based on the small signal value of emitter diode resistance, improves the design procedure and allows efficient Class A operation to be realized. Maximum available Class A output power was closely approached, but not fully attained with the methods reported in this paper.

INTRODUCTION

Of the different methods of oscillator analysis and design, one of the simplest and most general is that based upon a study of the dynamic admittance appearing at the output terminals of a two-port network.[1,2]

The basic oscillator circuit is shown in Fig. 1 with a transistor as the active device.

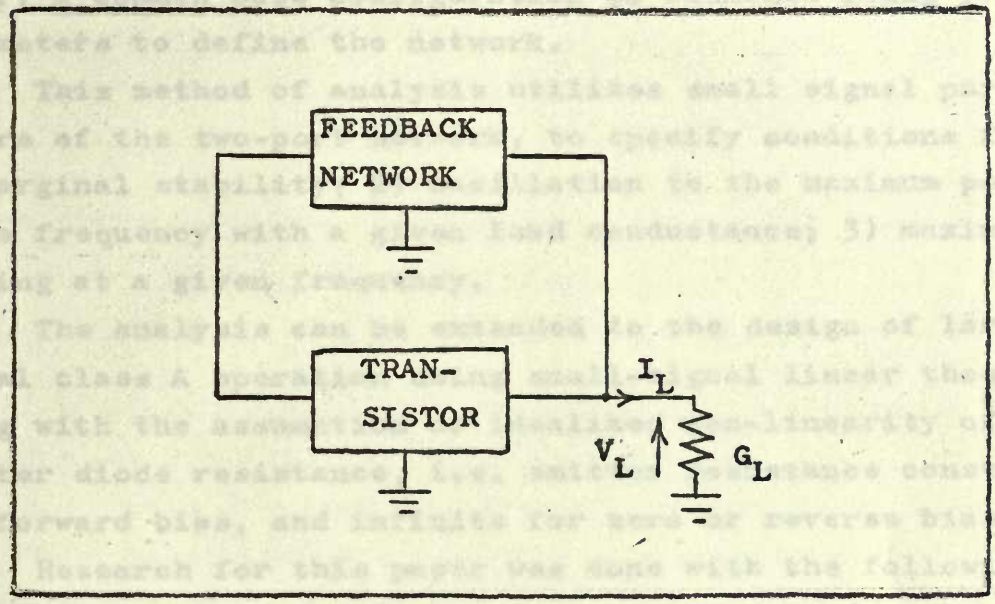

Fig. 1. Basic Oscillator Circuit

The output admittance that the network presents to the load can be expressed as a function of the transistor parameters and the feedback network parameters. This admittance for sinusoidal oscillation can be expressed as

$$Y_o = G_o + jB_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

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In order for oscillations to exist at a particular frequency the output susceptance must be approximately zero at that frequency, and the sum of the output conductance

and the load conductance must be negative, i.e.

$$B_o = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$G_L + G_o < 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

These conditions can be examined for any transistor configuration, using the network parameters for that particular configuration. Thus, this method of analysis is quite general with respect to configuration. In this paper, a common base configuration is examined using y-parameters to define the network.

This method of analysis utilizes small signal parameters of the two-port network, to specify conditions for 1) marginal stability; 2) oscillation to the maximum possible frequency with a given load conductance; 3) maximum loading at a given frequency.

The analysis can be extended to the design of large-signal class A operation using small-signal linear theory along with the assumption of idealized non-linearity of emitter diode resistance, i.e. emitter resistance constant for forward bias, and infinite for zero or reverse bias.

Research for this paper was done with the following objectives: 1) to verify the design procedure for the small signal case; 2) to examine the theory for extension to large signal oscillation and, if necessary, to base a modification of design procedure on experimental results.

SMALL SIGNAL THEORY

Since the y-parameters of a given transistor configuration are specified at a given frequency and bias point, the output admittance of the circuit of Fig. 2 is a function of load susceptance and the feedback network parameters.

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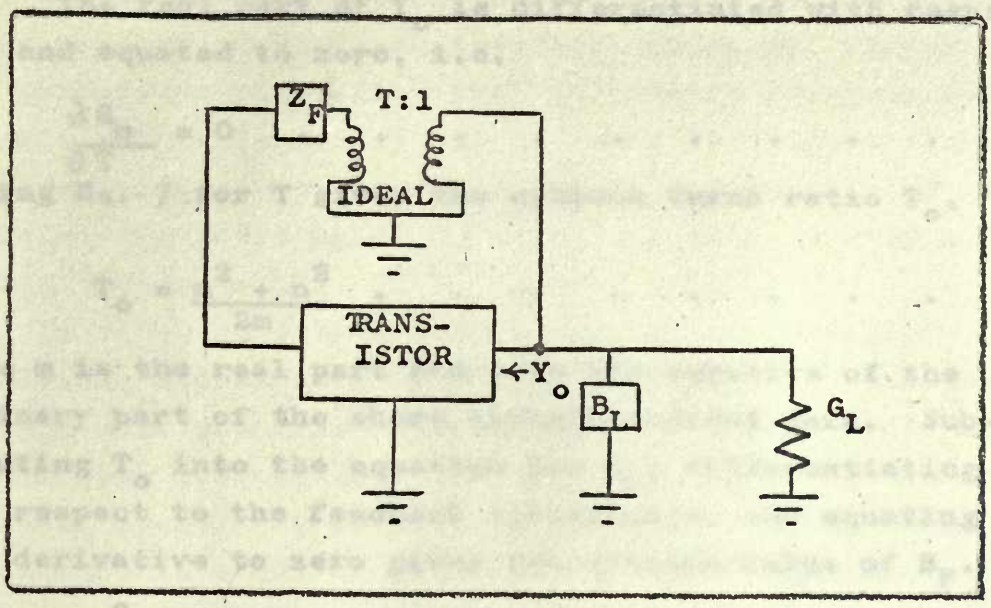

Fig. 2. Oscillator Circuit
with Transformer Feedback

An ideal transformer is assumed as the feedback element with a susceptance Z_F in series with the input terminal. This feedback network is a very basic form, from which other feedback networks such as the tee or pi can be derived. The output admittance presented to the load can be expressed in terms of the transformer turns ratio T, the susceptance B_F, and the y or h parameters. This output admittance (see Fig. 2) is

$$Y_o = h_{22} + \frac{(T - h_{12})\,(h_{21} + T)}{Z_F + h_{11}} \quad . \quad . \quad . \quad . \quad (4)$$

or in terms of y parameters

$$Y_o = y_{22} + \frac{T^2}{Z_F} + \frac{(y_{12} - T/Z_F)\,(y_{21} - T/Z_F)\,\frac{Z_F}{Y_{11}}}{Z_F + h_{11}} \quad (5)$$

If the output admittance is expressed in terms of its real and imaginary parts, it becomes

$$Y_o = G_n + jB_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

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An ideal transformer is assumed as the feedback element with a susceptance Z_p in series with the input terminal. This feedback network is a very basic form, from which other feedback networks such as the tee or pi can be derived. The output admittance presented to the load can be expressed in terms of the transformer turns ratio T, the susceptance B_p, and the y or h parameters. This output admittance (see Fig. 2) is

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$$Y_o = G_n + jB_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The real part of Y_o is differentiated with respect to T and equated to zero, i.e.

$$\frac{\partial G_n}{\partial T} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Solving Eq. 7 for T gives the optimum turns ratio T_o.

$$T_o = \frac{m^2 + n^2}{2m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where m is the real part and n is the negative of the imaginary part of the short circuit current gain. Substituting T_o into the equation for G_n, differentiating with respect to the feedback susceptance, and equating this derivative to zero gives the optimum value of B_F.

$$B_F^o = \frac{m}{nR' + mX'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where R' is the real and X' is the imaginary part of h_{11} and $h_{11} = 1/y_{11}$. $Y_F = G_F + jB_F$, the feedback admittance.

The optimum value of G_n is found from the above values of B_F^o and T_o.

$$G_n^o = \frac{1}{4g_{11}} \left[(g_{21} + g_{12})^2 + (b_{21} - b_{12})^2 \right] - g_{22} \quad . \quad (10)$$

where g_{nn} is the real part and b_{nn} is the imaginary part of y_{nn}.

Eq. 10 gives the maximum load that can be applied to the oscillator at the design frequency.

The same theory can be used to specify T and B_F for oscillation to the maximum frequency for a given load or to design for marginal stability.

MAXIMUM OUTPUT POWER

After having designed the circuit for maximum loading and applying the calculated maximum load, it is found

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After having designed the circuit for maximum loading and applying the calculated maximum load, it is found

that the output voltage and power in this load are quite small since the load impedance is small. Therefore, the assumption of small signal parameters is quite reasonable. Experimentally, it is found that increasing the load resistance increases the output power, but there is no sudden build up of oscillations and the output power is still very small.

In order to extend this design to large signal operation, approaching maximum efficiency, the collector voltage must saturate at the peak of one-half cycle and the emitter current must cut off at the peak of the other one-half cycle. It is assumed that the emitter diode resistance is equal to its small signal value until the current cuts off, at which point the resistance is assumed infinite. The current passing through the load conductance must cause a peak voltage equal to the collector supply voltage V_c. Since the mean bias current specifies the maximum collector current, the load conductance must be adjusted to give the desired voltage swing. With this value of G_L, the power dissipated is [3]

$$P_T = \frac{|\alpha| I_e V_c}{2} - \frac{I_e^2 R'}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

where $|\alpha|$ is the magnitude of alpha at design frequency; R' is the real part of h_{11}; I_e is the emitter current.

The first term of Eq. 11 is the available A-C power as the collector voltage swings over a range of 0 to V_c and the collector current varies from 0 to $|\alpha| I_e$. The second term is the power dissipated in the part of G_L which cancels the negative output conductance of the circuit, i.e. the power fed back to the input of the transistor.

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The ratio of these powers can be considered in order to solve for the adjusted value of G_L in terms of G_L^o to give maximum output power. This value is

$$G_L = \frac{4I_eR'}{|\alpha|V_c} G_L^o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Since the output voltage is greatly increased, the current that is fed back must be reduced to its original value; therefore the turns ratio must be reduced.

In order to keep the feedback current at its original value the turns ratio must be[3]

$$T_A = \frac{2I_eR'}{|\alpha|V_c} T_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where T_o is the turns ratio for maximum load conditions and T_A is the adjusted turns ratio.

Under the assumptions made in arriving at Eqs. (12) and (13), these adjusted values should give maximum power output; but since large signal operation is now in effect, rather than the assumed small signal operation, some modification in parameters might give more accurate results. It seems reasonable that the assumption of the emitter diode exhibiting extreme non-linearities is unrealistic and could be modified by using a more representative piece-wise linear approximation, so allowing a more accurate design procedure.

MEASUREMENT OF TRANSISTOR PARAMETERS

Since the parameters of the transistor play an important part in the analysis of the oscillator, accuracy in measuring these parameters is essential. For this reason, the parameters were measured by two different methods and compared to check the accuracy of the measurements.

The ratio of these powers can be considered in order to solve for the adjusted value of G_1 in terms of G_1^o to give maximum output power. This value is

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The first method of obtaining the transistor parameters utilized a Wayne Kerr admittance bridge. This bridge was used to measure the y parameters at the design frequency directly.

The second method was aimed at obtaining the equivalent circuit of the transistor by measurement of single element values. The circuit of Fig. 3 shows the equivalent circuit of the uniform base transistor.

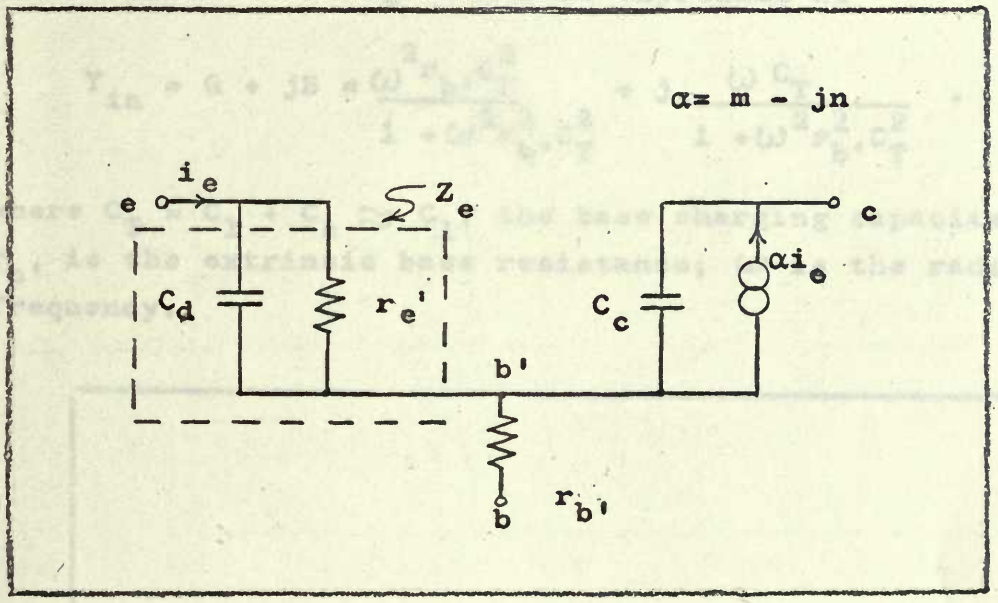

Fig. 3 Equivalent Circuit Representation of Uniform Base Transistor.

C_d is the diffusion capacitance.

$r_{e'}$ is the diode resistance.

$r_{b'}$ is the extrinsic base resistance.

C_c is the collector depletion region capacitance.

α is the short circuit current gain from emitter to collector.

The current gain as a function of frequency was measured and in this way the magnitude and phase of α at the design frequency was obtained.

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In order to obtain C_d and $r_{b'}$ the common emitter configuration was used. The output was short circuited. The input admittance was measured as the frequency was varied from about $\frac{1}{5}f_T$ to f_T. A low emitter current was used in order to obtain a high resistance in shunt with the diffusion capacitance. By neglecting this high resistance (compared to the impedance of the diffusion capacitance) the input admittance presented by the equivalent circuit of Fig. 4 can be expressed as

$$Y_{in} = G + jB = \frac{\omega^2 r_{b'} C_T^2}{1 + \omega^2 r_{b'}^2 C_T^2} + j\,\frac{\omega C_T}{1 + \omega^2 r_{b'}^2 C_T^2} \quad . \quad . \quad (14)$$

where $C_T = C_1 + C_c \approx C_1$, the base charging capacitance; $r_{b'}$ is the extrinsic base resistance; ω is the radian frequency.

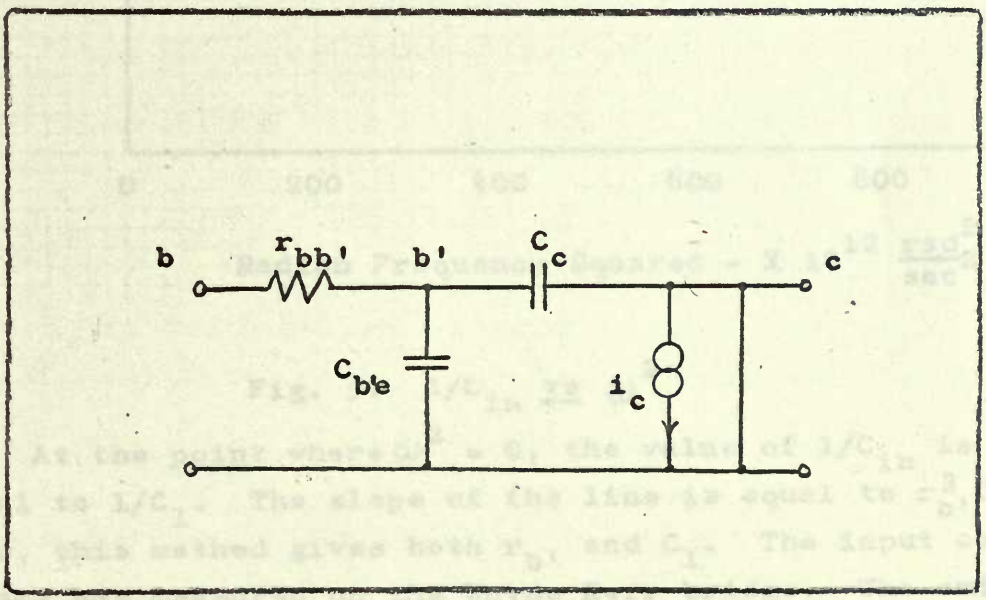

Fig. 4 High Frequency Equivalent Circuit of Transistor in Common Emitter Configuration for Low Emitter Control

In order to obtain C_d and r_b, the common emitter configuration was used. The output was short circuited. The input admittance was measured as the frequency was varied from about $\frac{1}{5}f_T$ to f_T. A low emitter current was used in order to obtain a high resistance in shunt with the diffusion capacitance. By neglecting this high resistance (compared to the impedance of the diffusion capacitance) the input admittance presented by the equivalent circuit of Fig. 4 can be expressed as

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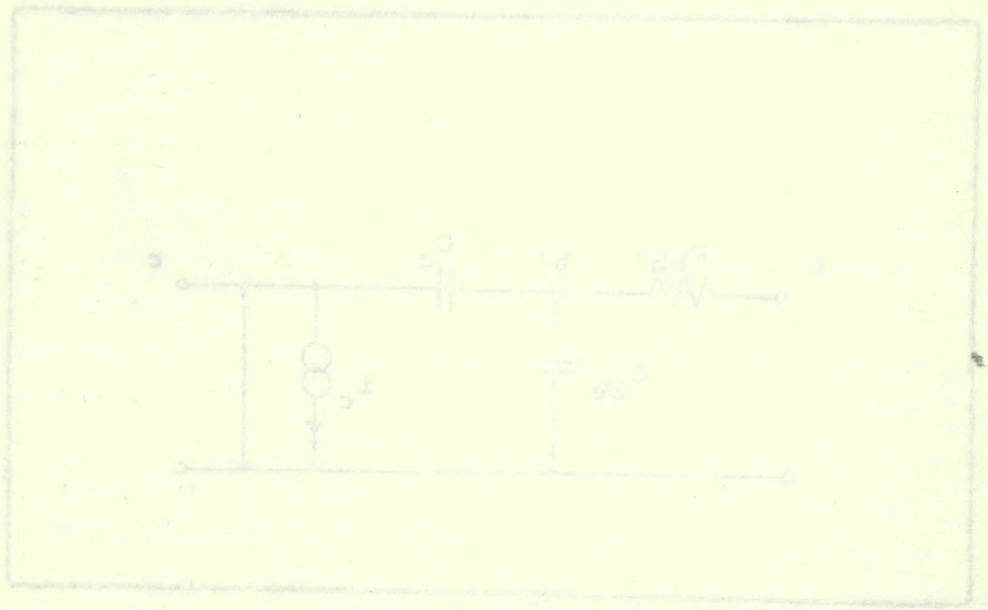

Fig. 4 High Frequency Equivalent Circuit of Transistor in Common Emitter Configuration for Low Emitter Control

From Eq. 14 the measured input capacitance is given by

$$C_{in} = \frac{B}{\omega} = \frac{C_1}{1 + \omega^2 r_{b'}^2 C_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and

$$1/C_{in} = \frac{1}{C_1} + \omega^2 r_{b'}^2 C_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

If $\frac{1}{C_{in}}$ is plotted as a function of ω^2, a straight line will result as in Fig. 5.

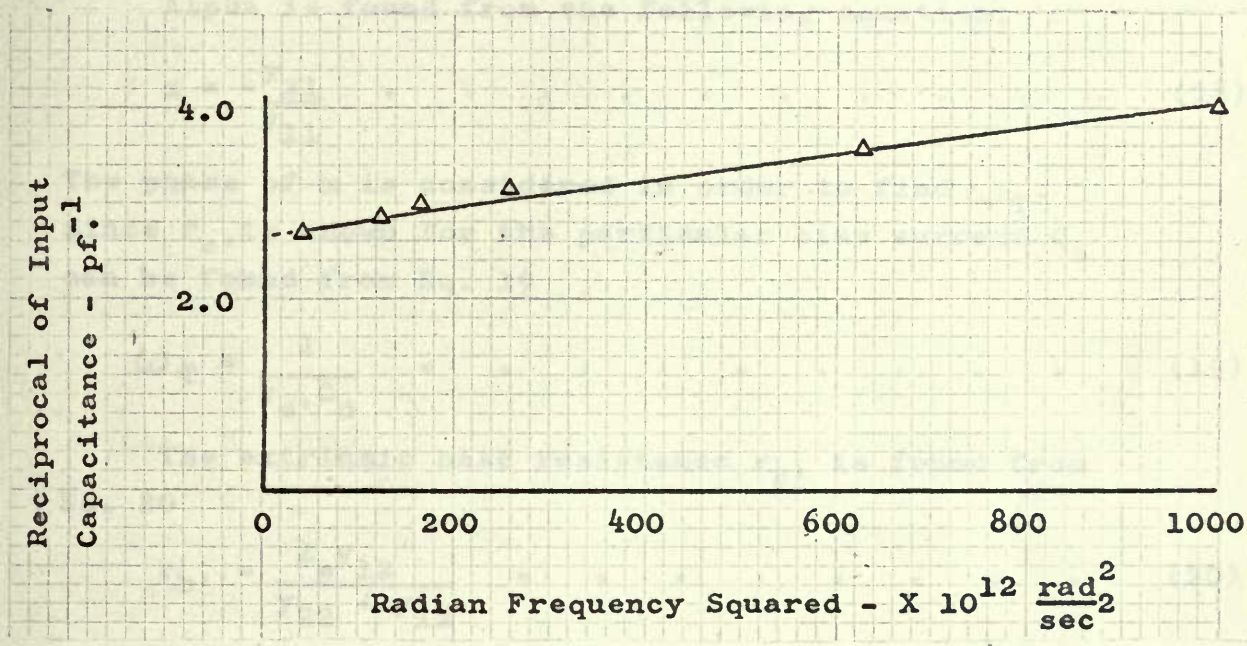

Fig. 5. $1/C_{in}$ vs ω^2

At the point where $\omega^2 = 0$, the value of $1/C_{in}$ is equal to $1/C_1$. The slope of the line is equal to $r_{b'}^2 C_1$. Thus, this method gives both $r_{b'}$ and C_1. The input capacitance was measured on the Wayne Kerr bridge. The emitter diffusion capacitance C_d in the common base configuration is found from[4]

$$C_d \approx \frac{2}{3} C_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

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$$C_d \approx \beta C_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The only element of the equivalent circuit remaining to be evaluated is C_c, the collector depletion layer capacitance. This can be measured by the Wayne Kerr bridge with the transistor in the common base configuration.

Since $r_{e'}$ is determined by the bias current, all element values of the equivalent circuit of Fig. 3 have now been determined. The equivalent circuit values can also be derived from the y parameters.

Alpha is found from the following equation:

$$\alpha = -\frac{y_{21}}{y_{11}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The phase of α is considered in order to find ω_T. Since $r_{e'}$ is known for the particular bias current C_e can be found from Eq. 19

$$\omega_T = \frac{1}{r_{e'} C_d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The extrinsic base resistance $r_{b'}$ is found from Eq. 20

$$r_{b'} = \frac{Z_e y_{12}}{y_{22} + y_{12}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where Z_e is the parallel combination of $r_{e'}$ and C_d.

The collector capacitance C_c is found from Eq. 21

$$C_c = j\frac{y_{12}}{\omega r_{b'} y_{11}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

which holds for $\frac{1}{\omega C_c} \gg r_{b'}$

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If the measured element values compare closely with those values derived from the y parameters, the y parameters can be assumed to have been measured accurately.

The equivalent circuit of the particular transistor used in this experiment was obtained from the two different methods reported above. The agreement was close enough to verify the accuracy of the measured values of the y parameters which are reported below, along with other pertinent data.

Transistor type IBM 075
Emitter current = 0.5ma

$y_{11} = 1.14\times10^{-2} - j\ .943\times10^{-2}$ $\quad y_{21} = -3.39\times10^{-3} + j\ .940\times10^{-2}$

$y_{12} = -2.04\times10^{-4} - j\ 4.40\times10^{-4}$ $\quad y_{22} = 4.35\times10^{-4} + j\ 6.28\times10^{-4}$

Element Values	Calculated (from y parameters)	Measured
C_d	575pf	580pf
$r_{e'}$	52Ω	55.5Ω
$r_{b'}$	59Ω	71Ω
C_c	24pf	22pf

VERIFICATION OF MAXIMUM INSTABILITY CASE

An oscillator was constructed based on the theory for the case of maximum load at a given frequency. The circuit is shown in Fig. 6 using the pi feedback network. The equations used in obtaining the pi or tee network from the transformer network are given in Sec. 1 of the appendix.

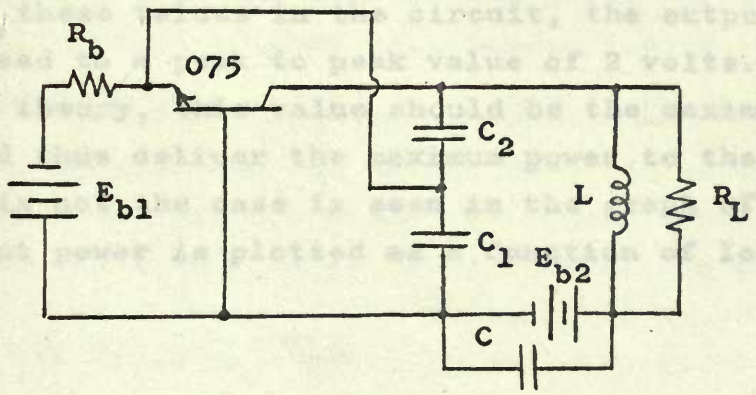

Fig. 6 Oscillator Designed for Maximum Loading

C_1 = 176pf $\quad$ L = 5μh-9μh adjustable

C_2 = 246pf $\quad$ R_b = 10,000 ohms

The equivalent circuit of the particular transistor used in this experiment was obtained from the two different methods reported above. The agreement was close enough to verify the accuracy of the measured values of the y parameters which are reported below, along with other pertinent data.

Transistor type IBM 075
Emitter current = 0.5ma

$Y_{11} = 1.14 \times 10^{-2} - j\ .943 \times 10^{-2}$ $\quad Y_{21} = -3.39 \times 10^{-3} + j\ .940 \times 10^{-2}$

$Y_{12} = -2.04 \times 10^{-4} - j\ 4.40 \times 10^{-4}$ $\quad Y_{22} = 4.35 \times 10^{-4} + j\ 6.28 \times 10^{-4}$

Element Values	Calculated (from y parameters)	Measured
C_d	575pf	580pf
r_e,	52Ω	55.5Ω
r_b,	59Ω	71Ω
C_c	24pf	22pf

VERIFICATION OF MAXIMUM INSTABILITY CASE

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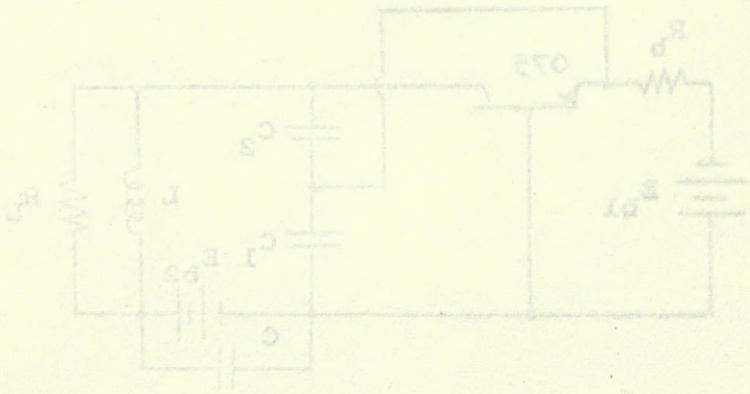

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The oscillator was designed to operate at 5MC/s. The minimum load resistance was calculated to be 513 ohms at this frequency. With an infinite R_L or an open circuit output, the circuit oscillated at 5.2 MC/S. The output voltage was 1.3 volts peak to peak. As the circuit was loaded the frequency approached 5 MC/S. Using a load resistance of 510 ohms, the circuit oscillated at exactly 5 MC/S. Any increase of load conductance stopped the oscillation completely. This value of conductance then gives the maximum load that can be applied to the output terminals without stopping the oscillations.

The power delivered to the load under these conditions was quite small; the output voltage was only .05 volts peak to peak.

In order to increase the output power, the small signal theory was extended to calculate new values for the feedback network and the load conductance (Eqs. 12 and 13). These calculations are found in Sec. 2 of the appendix.

The revised values in the feedback network are as follows: $C_1 = 420pf$, $C_2 = 2.16pf$ and $L = 20\mu h$. The optimum load resistance for maximum power output was calculated to be 21,000 ohms.

With these values in the circuit, the output voltage was increased to a peak to peak value of 2 volts. According to the theory, this value should be the maximum output voltage and thus deliver the maximum power to the load. That this is not the case is seen in the graph of Fig. 7 where output power is plotted as a function of load resistance.

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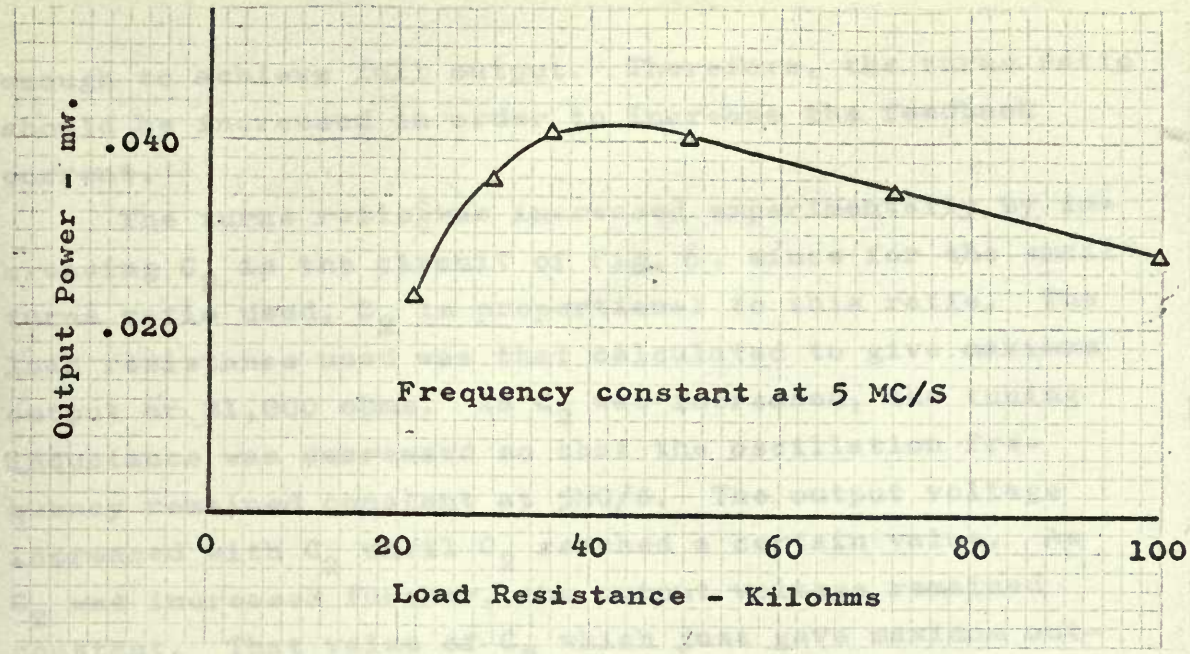


Fig. 7 Output Power vs Load Resistance for Circuit Based on Small Signal Parameters

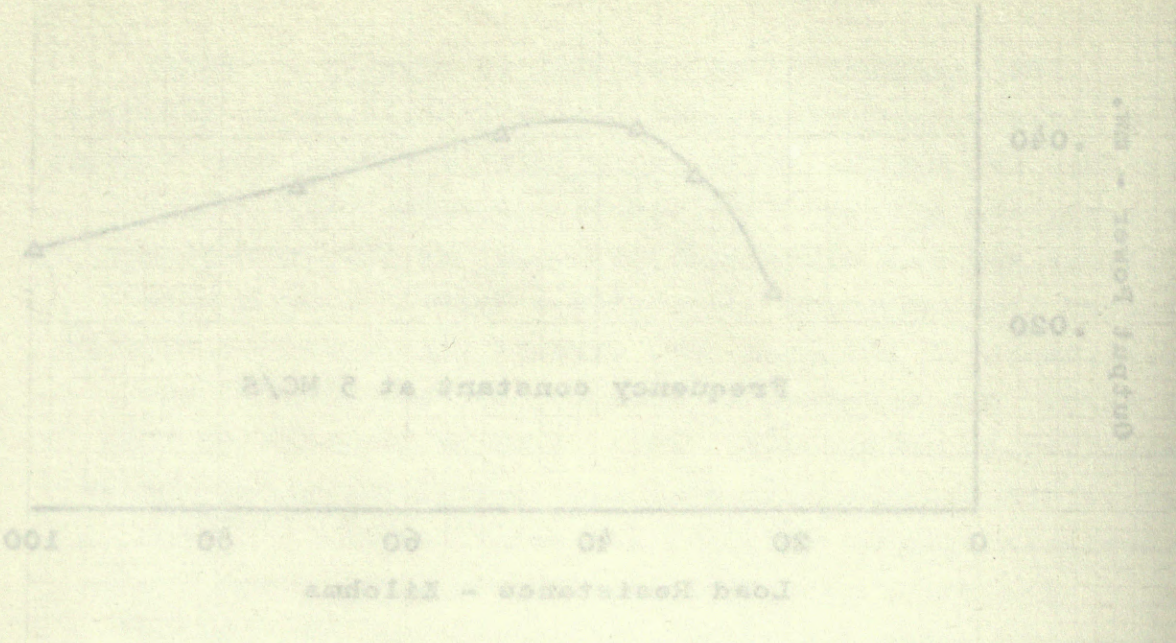
The power reaches a peak of .042 mw. when the load resistance is approximately 36,000 ohms. This peak power is still far short of the .856 mw. that the oscillator is capable of developing.

These results demonstrate that the theory used does not predict the correct values to be used in the feedback network.

The next step is to determine the procedure to be used in order to make the peak of the power curve occur at the calculated value of load resistance, i.e. 21,000 ohms.

MODIFICATION OF DESIGN PROCEDURE TO OBTAIN MAXIMUM POWER OUTPUT

As the emitter current swings from its maximum value to zero value, the emitter diode resistance increases from a small value to a very large value. Since the current that is fed back depends on the turns ratio which was calculated on the basis of a constant, relatively small emitter resistance, it could be expected that this current is not



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MODIFICATION OF BASIC PROCEDURE TO OBTAIN MAXIMUM POWER OUTPUT

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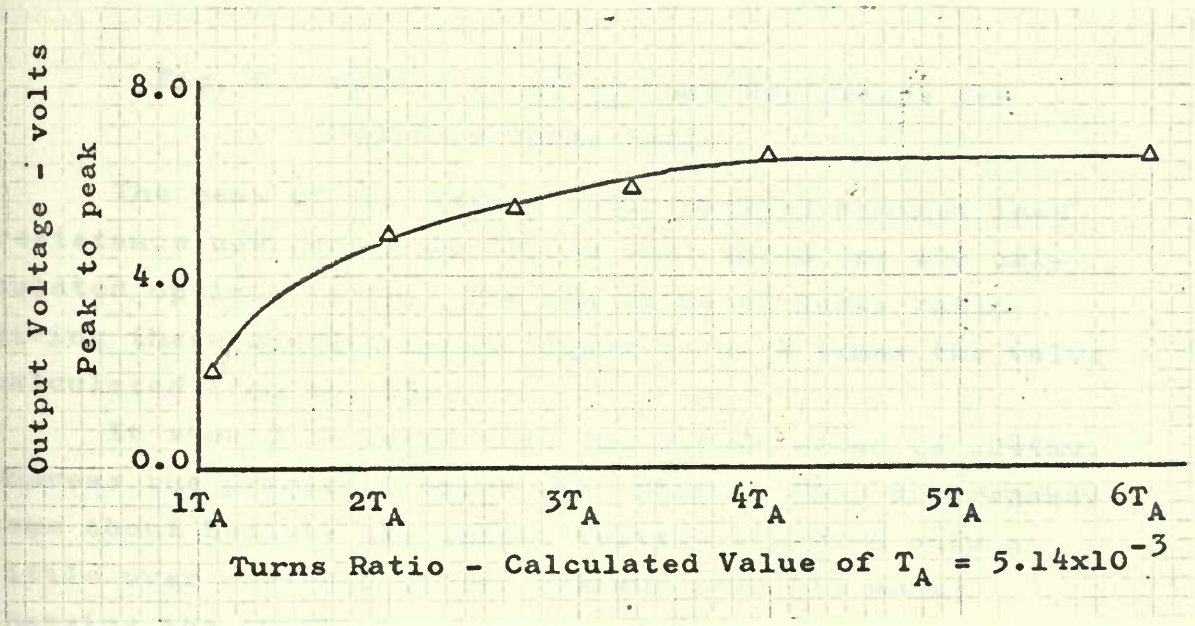
enough to achieve full output. Therefore, the turns ratio should be increased in order to increase the feedback current.

The turns ratio was increased experimentally by increasing C_2 in the circuit of Fig. 6, since for the small turns ratio used, C_2 is proportional to this ratio. The load resistance used was that calculated to give maximum output or 21,000 ohms. As C_2 was increased, the tuning inductance was decreased so that the oscillation frequency remained constant at 5MC/S. The output voltage increased with C_2 until C_2 reached a certain value. As C_2 was increased further, the output voltage remained constant. That value of C_2 which just gave maximum output was chosen as the proper value.

The graph of Fig. 8 shows a plot of output voltage vs turns ratio.



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data are shown in the graph of Fig. 9.

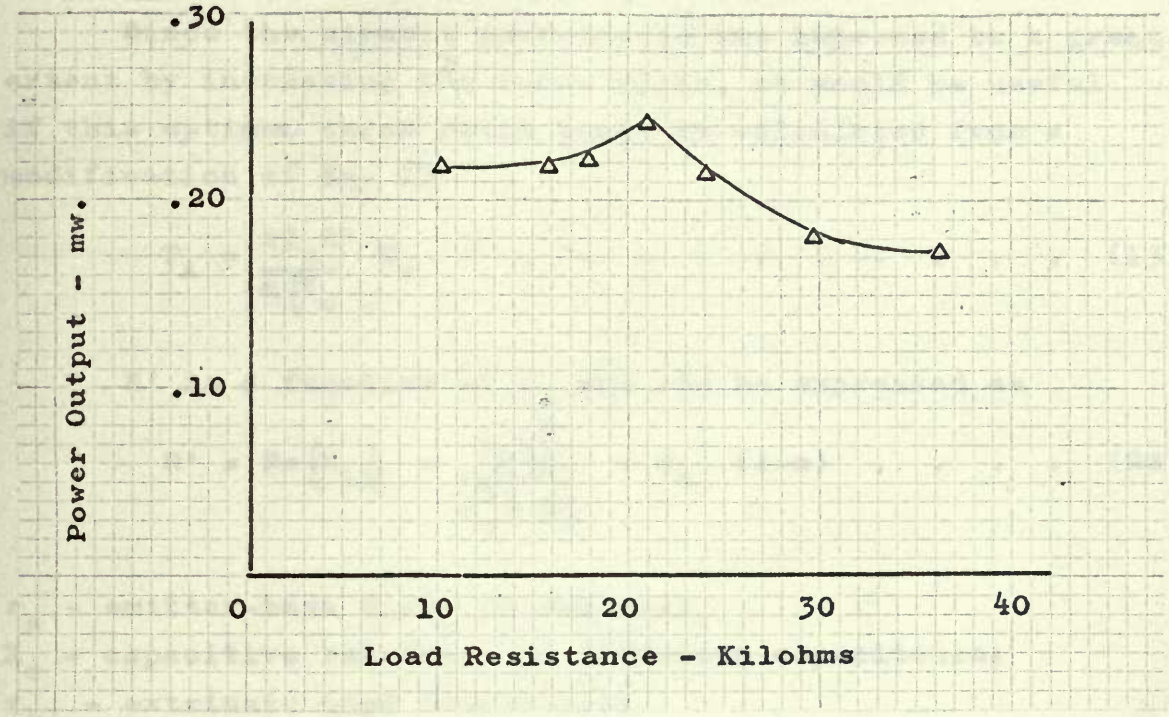


Fig. 9. Output Power vs Load Resistance for Modified Turns Ratio.

The peak of the output power plotted against load resistance now occurs at 21,000 ohms which was the calculated optimum value. The new value of turns ratio giving these conditions was found to be 4 times the value calculated from eq. 13.

It should be noted that the output power is .244mw. whereas the predicted value was .856mw. This discrepancy came about because the output voltage achieved only a little over one-half of its maximum possible swing. Changing the turns ratio appears to have improved two aspects of circuit performance. First, the power output is much greater than the value achieved before the turns ratio was changed. Second, the power output, which varies

ith the load resistance, reaches a peak at that value
f resistance calculated from Eq. 12.

Since the circuit performance was improved to a great
xtent by increasing the turns ratio, it would be useful
f this optimum turns ratio could be calculated from a
odification of Eq. 13

$$T_A = \frac{2I_e R'}{|\alpha| V_c} T_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

R' is a function of r_e, and can be expressed as

$$R' = Re\{h_{11}\} = \frac{r_e X_c^2}{r_e^2 + X_c^2} + r_b, (1-m) \quad . \quad . \quad . \quad . \quad (22)$$

r_e, - emitter-base diode resistance;
X_c - capacitive reactance of diffusion capacitance;
r_b, - extrinsic base resistance;
m - real part of short circuit current gain.

Since r_e, and X_c both vary in the same manner as the emitter current is changed, the first term varies directly as r_e, As the emitter current approaches zero, the first term of Eq. 22 becomes large compared to r_b,(1-m). Therefore, the simplifying assumption that R' varies directly as r_e, is not unreasonable.

From the experimental results, the turns ratio T_A had to be multiplied by a factor of four to get near optimum performance. This indicates that if the measured value of R' under small signal conditions is multiplied by a factor of four, the result will be more representative of the large signal value of R'. This increased value of R' will then be reflected in Eq. 13 to give the increased value of T_A.

The graph of Fig. 10 shows the emitter diode characteristic of the transistor. A straight line is drawn tangent to the curve at 0.5ma. The reciprocal of the slope of this line gives the small signal value of the diode resistance for a bias of 0.5ma. Also drawn through the 0.5ma point is the line whose slope gives the piece-wise linear approximation of diode conductance over the full current swing from 0 to 1 ma. The slope of this line is equal to one fourth of the slope of the first line, reflecting the fact that the large signal value of $r_{e'}$ is approximately four times the small signal value.

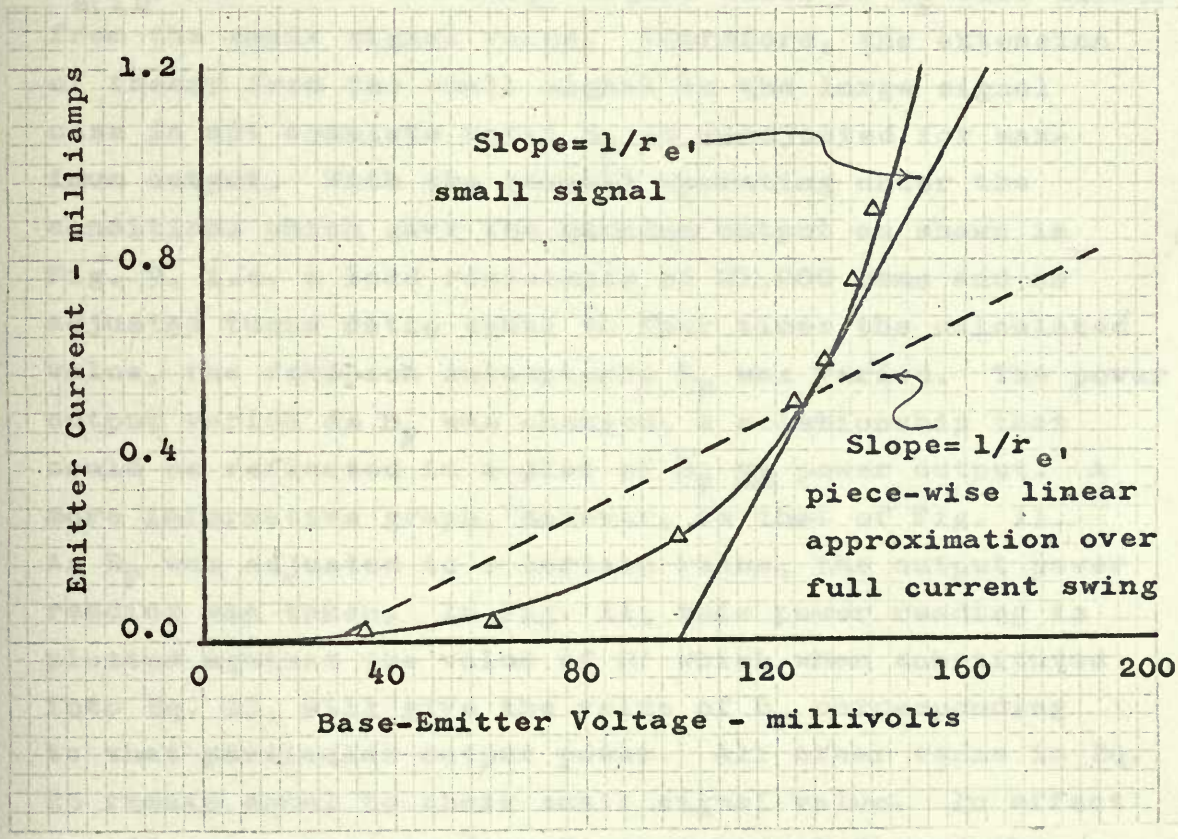


Fig. 10. Emitter Diode Characteristic

ADJUSTING THE PHASE OF THE FEEDBACK CURRENT

Since the large signal value of r_e, has increased over the small signal value, the phase of the feedback current will be different than in the small signal design case. In the circuit of Fig. 2, the phase of the feedback current is adjusted by the susceptance B_F. This susceptance can be calculated from Eq. 23[2]

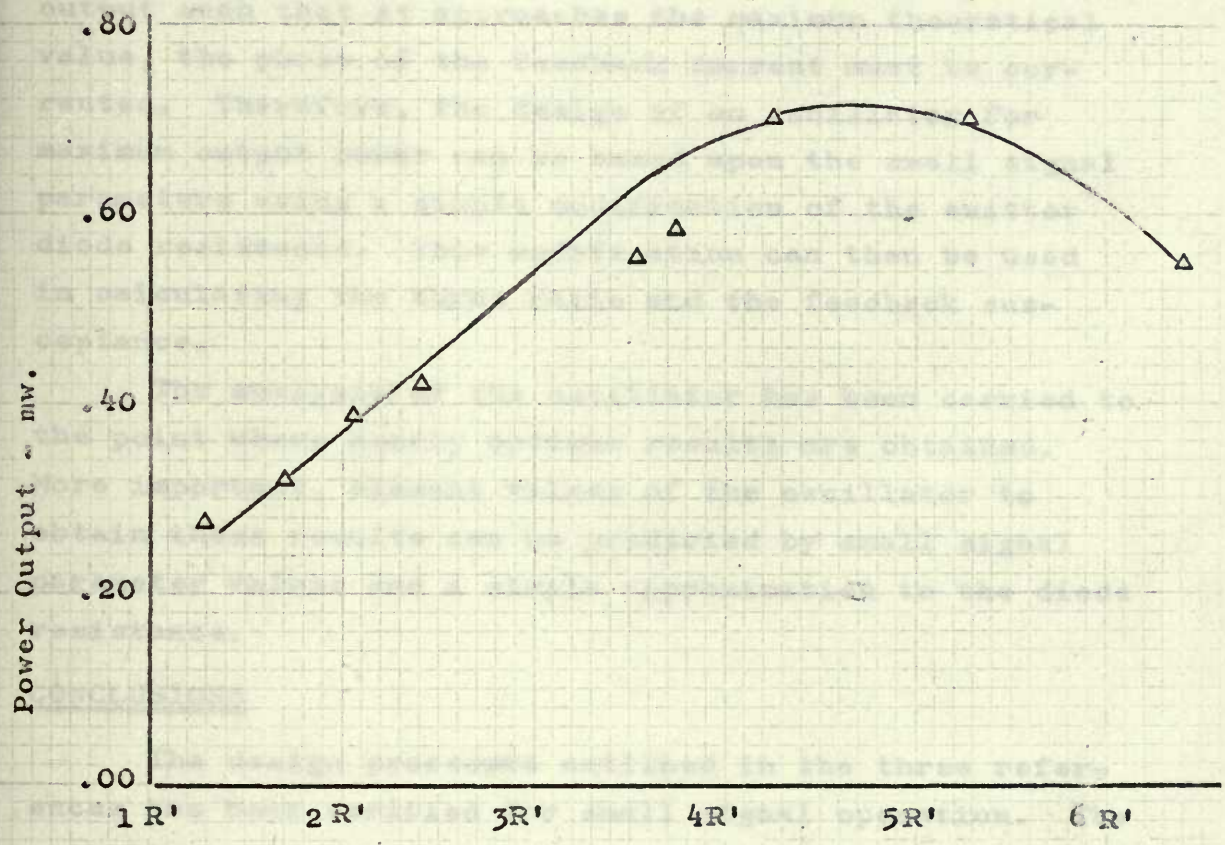
$$B_F = \frac{m}{nR' + mX'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (23)$$

where $\alpha = m - j\,n$ and $h_{11} = R' + jX'$

It is seen that B_F depends on R', which is also equal $R_e\{h_{11}\}$, and hence the large signal value of B_F will differ from the small signal value. Therefore, the extension of theory from the small signal to the large signal case is not complete until B_F is readjusted for maximum output. With the circuit operating under the conditions which gave the maximum output as shown in Fig. 9, i.e. a load resistance of 21,000 ohms and an adjusted turns ratio equal to four times the calculated value, the feedback susceptance B_F was varied. The power output varied as B_F was changed, a relationship that could be reflected in a plot of B_F vs power output. A more informative graph, however, is that of Fig. 11. As B_F was adjusted to a certain value, the output power reading was taken. In Fig. 11, this power reading is plotted against the value of R' which when substituted into Eq. 23, will give the value of B_F corresponding to that particular output power. All other terms in Eq. 23 remain equal to their small signal value. In effect Eq. 23 was solved to find R' in terms of B_F and the remaining terms.

$$R' = \frac{m/B_F - mX'}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

The power output for a given B_F can then be plotted against R' as in Fig. 11. In this way, the role of R' in optimizing the phase of the feedback current for maximum power output can be seen.



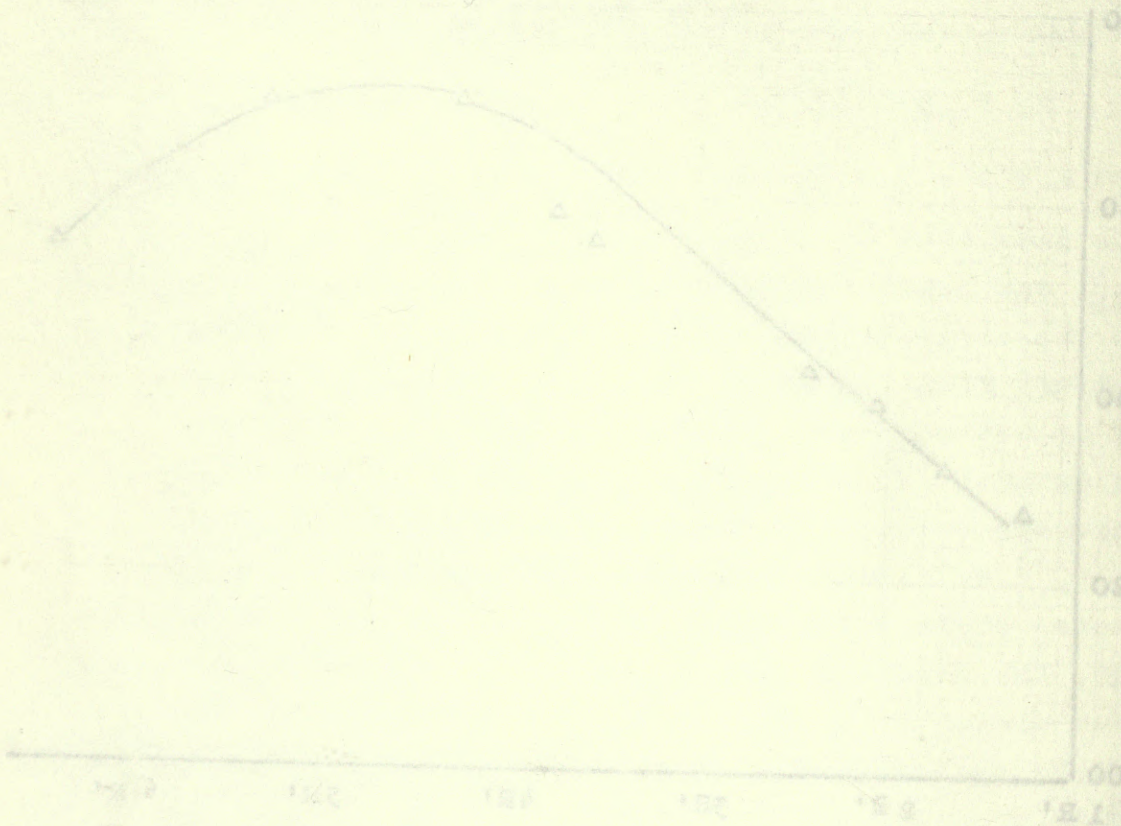
Resistance Used In Calculating B_F from Eq. 23 - R'

R' = small signal value of Re$\{h_{11}\}$

Fig. 11. Power Output vs R'

Fig. 11 shows that the proper phase for maximum output is obtained when B_F is calculated on the basis of R' being about 4.4 times the small signal value.

The power output for a given γ can then be plotted against R' as in Fig. 11. In this way, the ratio of R' in optimizing the phase of the feedback current for maximum power output can be seen.



Relations Used In Calculating R_p from $\Delta\gamma = \gamma - \gamma'$

γ' = small signal value of $\ln\{M_{11}\}$

Fig. 11. Power Output $\gamma = \gamma'$

Fig. 11 shows that the proper phase for maximum output is obtained when R_p is calculated on the basis of γ' being about 4.4 times the small signal value.

The above result tends to substantiate the result of the last section; i.e. the large signal value of R' is approximately four times the small signal value.

It is seen that correcting the turns ratio makes the peak power output occur at the proper value of load resistance. However, in order to increase the power output such that it approaches the maximum theoretical value, the phase of the feedback current must be corrected. Therefore, the design of an oscillator for maximum output power can be based upon the small signal parameters using a simple modification of the emitter diode resistance. This modification can then be used in calculating the turns ratio and the feedback susceptance.

The analysis of the oscillator has been carried to the point where nearly optimum results are obtained. More important, element values of the oscillator to obtain these results can be predicted by small signal parameter values and a simple approximation to the diode resistance.

CONCLUSIONS

The design procedure outlined in the three references has been verified for small signal operation. The extension to large signal operation gives results which are not optimum. An important reason for these non-optimum conditions is the fact that the emitter diode resistance varies appreciably over the oscillation cycle. A piece-wise linear approximation of this resistance, which is more representative of the large signal value, was obtained. This value was found to be about four times the small signal value. Using this modification of diode resistance, the output power was

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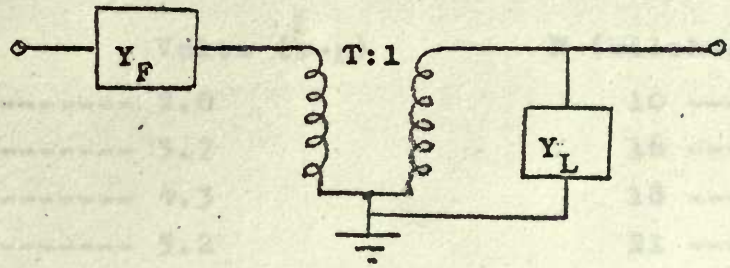
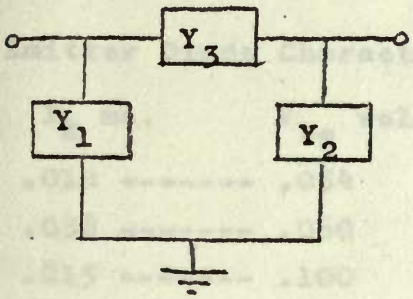
82% of the maximum theoretical output. It should be profitable to continue this research with the objective of obtaining the maximum available output power.

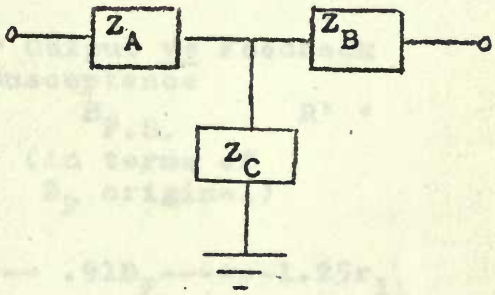
A P P E N D I X

Section 1. Equations for obtaining equivalent feedback circuits from basic transformer feedback.



a) Transformer - coupled form





b) Pi impedance - coupled form

$Y_1 = Y_F(1 - T)$

$Y_2 = Y_L - TY_F(1 - T)$

$Y_3 = TY_F$

c) Tee impedance - coupled for

$Z_A = Z_F - TZ_L(1 - T)$

$Z_B = Z_L(1 - T)$

$Z_C = TZ_L$

Section 2. Calculations of element values for maximum output.

$$G_L = \frac{4I_eR'}{|a|V_c} \qquad G_{LO} = \frac{4(.5 \times 10^{-3})(52.5)}{.715\ (6)}\ 1.95 \times 10^{-3} = 4.47 \times 10^{-5}$$

$$R_L = 1/G_L = 21{,}000 \text{ ohms}$$

$$T_A = \frac{2I_eR'}{|a|V_c}\ T_o = \frac{2\ (.5 \times 10^{-3})(52.5)}{.715\ (6)}\ .42 = 5.14 \times 10^{-3}$$

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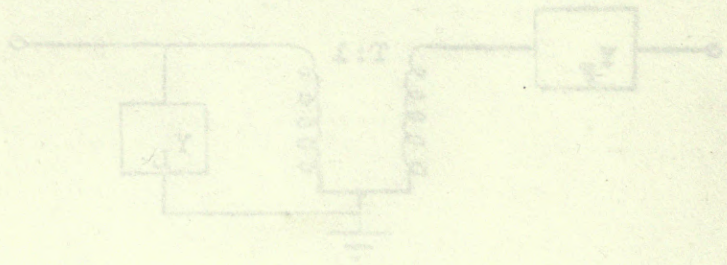

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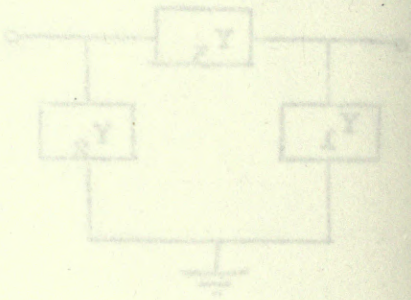

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Section 3. Experimental Data

Changing T_A with $R_L = 21^K$

T_A	Volts (p-p)
1.0	2.0
1.3	3.7
1.8	4.3
2.7	5.2
3.5	5.6
4.0	6.4
6.0	6.4

Changing R_L with $T_A^o = 4T_A$

R (kilohms)	Volts (p-p)
10	4.4
16	5.3
18	5.7
21	6.4
24	6.4
30	6.6
36	7.0

Emitter Diode Characteristic

I_E ma.	V_{be} volts
.012	.034
.038	.060
.215	.100
.400	.116
.500	.122
.600	.130
.780	.135
.900	.140

Power Output vs Feedback Susceptance

P_{out} mw.	$B_{F.B.}$ (in terms of B_F original)	R' *
.27	$.91B_F$	$1.25r_{11}$
.33	$.81B_F$	$1.70r_{11}$
.38	$.70B_F$	$2.10r_{11}$
.43	$.63B_F$	$2.40r_{11}$
.56	$.51B_F$	$3.50r_{11}$
.60	$.47B_F$	$3.70r_{11}$
.70	$.42B_F$	$4.20r_{11}$
.70	$.35B_F$	$5.25r_{11}$
.64	$.30B_F$	$6.50r_{11}$

* R' is found from eq. 23. It is the value which when substituted into eq. 23 will give the corresponding value of $B_{F.B.}$.

Section 3. Experimental Data

Changing T_A with $R_L = 22^k$

T_A	Volts (p-p)
1.0	2.0
1.3	3.7
1.8	4.3
2.7	5.2
3.5	5.6
4.0	6.4
6.0	6.4

Changing R_L with $T_A^o = 4T_A$

R_L (kilohms)	Volts (p-p)
10	4.4
16	5.3
18	5.7
21	6.4
24	6.4
30	6.6
36	7.0

Emitter Diode Characteristic

I_E ma.	V_{be} volts
.012	.034
.038	.060
.215	.100
.400	.116
.500	.122
.600	.129
.780	.135
.900	.140

Power Output vs Feedback Susceptance

P_{out} mw.	B_F (in terms of B_F original)
.27	.918
.33	.818
.38	.700
.43	.635
.56	.515
.60	.473
.70	.420
.79	.351
.84	.308

B_F is taken from the value which will give the corresponding value of P_{out}.

R E F E R E N C E S

1. D.F. Page and A.R. Boothroyd, "Instability in Two-Port Active Networks," IRE Trans. on Circuit Theory, vol. CT-5, No. 3; June 1958.

2. D.F. Page, "A Design Basis for Junction Transistor Oscillator Circuits," Proc. IRE, vol. 46, pp 1271-1280, June 1958.

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